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AN INTENSE TURBULENT THERMIC IN A STABLY STRATIFIED ATMOSPHERE. NUMERICAL MODELING

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Nonsteady-state convective turbulent flow due to climbing of a volume of liquid or gas with a deficit of density (a thermic) in an unrestricted medium has been theoretically studied in a number of works, which are reviewed in [1, 2]. These investigations allow one to qualitatively describe the gas-dynamic structure of the flow and the mechanism behind heat and mass exchange between the thermic and the surrounding medium. A study of these flows is complicated by the lack of data on the intensity of turbulent exchange in a thermic, which leads to arbitrariness in selecting the values for the coefficients of turbulent transfer. Here, the conditions required for adequate numerical modeling of a turbulent thermic are determined, and the dynamics of its climb from the moment of formation to that of height equilibrium in a stably stratified atmosphere are calculated.

1. Formulation of the Problem. The system of turbulence equations for describing axially symmetric, nonsteady-state convective flow of gas in a heat-concentrated thermic has the following form in the Boussinesq approximation:

$$\frac{\partial\Omega}{\partial t}+\frac{\partial}{\partial x}\frac{\Omega}{r}\frac{\partial\psi}{\partial r}-\frac{\partial}{\partial r}\frac{\Omega}{r}\frac{\partial\psi}{\partial x}=\frac{\partial}{\partial x}E\frac{\partial\Omega}{\partial x}+\frac{\partial}{\partial r}\frac{E}{r}\frac{\partial\Omega r}{\partial r}-\frac{\partial\omega}{\partial r},$$

$$\frac{\partial\omega}{\partial t}+\frac{\partial}{\partial x}\frac{\omega}{r}\frac{\partial\psi}{\partial r}-\frac{1}{r}\frac{\partial}{\partial r}\omega\frac{\partial\psi}{\partial x}=\mathrm{Pr}^{-1}\left(\frac{\partial}{\partial x}E\frac{\partial\omega}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}Er\frac{\partial\omega}{\partial r}\right)-\frac{N^2}{r}\frac{\partial\psi}{\partial r}, \quad (1.1)$$

$$\frac{\partial\vartheta}{\partial t}+\frac{\partial}{\partial x}\frac{\vartheta}{r}\frac{\partial\psi}{\partial r}-\frac{1}{r}\frac{\partial}{\partial r}\vartheta\frac{\partial\psi}{\partial x}=\mathrm{Pr}^{-1}\left(\frac{\partial}{\partial x}E\frac{\partial\vartheta}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}Er\frac{\partial\vartheta}{\partial r}\right)-\frac{N^2}{g\beta r}\frac{\partial\psi}{\partial r},$$

$$\frac{\partial\varepsilon}{\partial t}+\frac{\partial}{\partial x}\frac{\varepsilon}{r}\frac{\partial\psi}{\partial r}-\frac{1}{r}\frac{\partial}{\partial r}\varepsilon\frac{\partial\psi}{\partial x}=\mathrm{Sc}^{-1}\left(\frac{\partial}{\partial x}E\frac{\partial\varepsilon}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}Er\frac{\partial\varepsilon}{\partial r}\right),$$

$$\Omega=\frac{1}{r^2}\frac{\partial\psi}{\partial r}-\frac{1}{r}\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial^2\psi}{\partial x^2},\quad \omega=g\frac{\rho_a-\rho}{\rho}\approx g\left[\beta\vartheta+\varepsilon\left(\frac{\mu_a}{\mu}-1\right)\right],$$

$$\Omega=\psi=\frac{\partial\omega}{\partial r}=\frac{\partial\vartheta}{\partial r}=\frac{\partial\varepsilon}{\partial r}=0,\ r=0,\ \Omega=\psi=\omega=\vartheta=\varepsilon\to 0,\ r^2+x^2\to\infty.$$

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Here x and r are the vertical and radial coordinates; Ω, component of the vorticity vector perpendicular to the plane (x, r); ψ, flow function; g, projection of the acceleration due to gravity onto the axis x; $\vartheta = T - T_a$, $\varepsilon = Y - Y_a$, excess temperature and mass of a gas concentration with a molecular weight of μ relative to the surrounding medium; $\beta \approx 1/T_a$, thermal expansion coefficient; $N^2 = g\beta(dT_a/dx + g/c_p)$, Väisälä–Brunt coefficient, which characterizes the degree of atmospheric stability; and Pr and Sc, Prandtl and Schmidt turbulence numbers (we will assume that $Pr \equiv Sc$).

The turbulence viscosity E in a freely convective flow is due to motion whose characteristics are determined by the integral quantity for the reserve buoyancy of the climbing volume of gas $\Pi_0 = \int_{-\infty}^{\infty}\int_0^{\infty} \omega r dr dx$, m^4/sec^2. Then one can write $E = \Pi_0^{1/2}\nu$, where ν in the general case depends on the spatial coordinates of the turbulence coefficient. This approach for assigning the turbulence viscosity is principally different from that in [3, 4], where E is assumed independent of the motion parameters, i.e., one uses the analogy between laminar and turbulent flow.

Observations of thermics under various conditions indicate that, as a rule, from some moment in time after the initiation of motion, the climb of the buoyant cloud enters into a state of self-similarity, which is characterized by time-similarity of the cloud form and by a flow that is independent of the actual initial conditions [5]. Considering this, a study of the climb of a thermic into a stably stratified atmosphere can be divided into three stages: an initial stage (which determines the parameters Π_0 and R_0, the characteristic initial dimensions of the cloud), a self-similarity stage Π_0, and a hovering stage Π_0, N. The hovering stage, where the cloud climbs into a stably stratified medium, begins at times comparable with the characteristic time for thermal restructuring of the atmosphere ($t > N^{-1}$). When $t \ll N^{-1}$, the thermic "feels" the state of the surrounding medium and moves accordingly in a uniformly stratified atmosphere ($dT_a/dx = -g/c_p$, $N = 0$). The self-similarity state is achieved for $R_0^2\Pi_0^{-1/2} \ll t \ll N^{-1}$, when the thermic has already "forgotten" the conditions in which it is formed (its volume is many times greater than the initial volume), but it still does not "feel" the conditions of the surrounding medium. It is useful to render each function dimensionless in each stage for obtaining the most general results.

2. Self-Similarity Stage in the Climb of a Thermic. It is evident from (1.1) that the dynamic problem of the climb of a heat-concentrated thermic can be solved on the basis of only the equations of motion and the equations for the acceleration of buoyant forces ω without detailing the thermal and density distributions. Rendering the dynamic problem for the self-similarity stage in the climb of the thermic into dimensionless form, we find that

$$\begin{gathered} \Omega = t^{-1}W(\eta, \zeta), \qquad E = \Pi_0^{1/2}\nu, \\ \psi = \Pi_0^{3/4}t^{1/2}F(\eta, \zeta), \quad \eta = r\Pi_0^{-1/4}t^{-1/2}, \\ \omega = \Pi_0^{1/4}t^{-3/2}\varphi(\eta, \zeta), \quad \zeta = x\Pi_0^{-1/4}t^{-1/2}. \end{gathered} \tag{2.1}$$

Because of the absence of data on the distribution of turbulence, viscosity in a thermic, a simple model for isotropic turbulence is used where the quantity ν is assumed independent of the spatial coordinates.

The dimensionless system of equations for the self-similarity stage in the climb of the thermic is obtained after substituting (1.1) into (2.1) for N = 0

$$\frac{\partial}{\partial\zeta} W\left(\frac{\zeta}{2} - \frac{1}{\eta}\frac{\partial F}{\partial\eta}\right) + \frac{\partial}{\partial\eta} W\left(\frac{\eta}{2} + \frac{1}{\eta}\frac{\partial F}{\partial\zeta}\right) + \nu\left(\frac{\partial^2 W}{\partial\zeta^2} + \frac{\partial}{\partial\eta}\frac{1}{\eta}\frac{\partial W\eta}{\partial\eta}\right) - \frac{\partial\varphi}{\partial\eta} = 0, \tag{2.2}$$

$$\frac{\partial}{\partial\zeta}\varphi\left(\frac{\zeta}{2} - \frac{1}{\eta}\frac{\partial F}{\partial\eta}\right) + \frac{1}{\eta}\frac{\partial}{\partial\eta}\varphi\eta\left(\frac{\eta}{2} + \frac{1}{\eta}\frac{\partial F}{\partial\zeta}\right) + \mathrm{Pr}^{-1}\nu\left(\frac{\partial^2\varphi}{\partial\eta^2} + \frac{\partial^2\varphi}{\partial\zeta^2} + \frac{1}{\eta}\frac{\partial\varphi}{\partial\eta}\right) = 0,$$

$$\frac{\partial^2 F}{\partial\eta^2} + \frac{\partial^2 F}{\partial\zeta^2} - \frac{1}{\eta}\frac{\partial F}{\partial\eta} + \eta W = 0, \quad \int_{-\infty}^{\infty}\int_0^{\infty}\varphi\eta\, d\eta\, d\zeta = 1,$$

$$W = F = \frac{\partial\varphi}{\partial\eta} = 0, \ \eta = 0; \ W = F = \varphi \to 0, \ \eta^2 + \zeta^2 \to \infty.$$

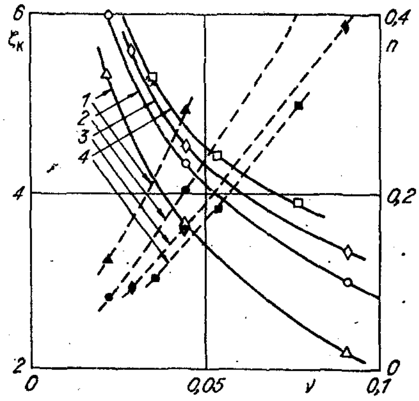

Fig. 1

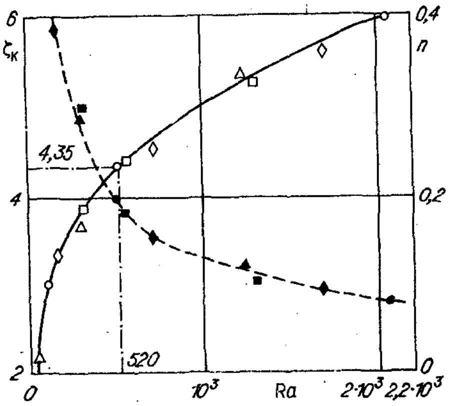

Fig. 2

The unknown constants ν and Pr enter into (2.2). For finding them, the problem for a self-similar thermic is solved for various values of ν and Pr to obtain the dependence of these parameters on those characteristics of the flow that are permitted by experimental measurements and to find the values of the unknown constants from a comparison of the calculation results and the experimental data. Problem (2.2) is solved by using an implicit scheme for the direction variables. A detailed description of the difference scheme is given in [2].

Calculated dependences of the kinematic characteristics of the climb of the thermic are represented in Fig. 1, where the coordinates of its upper border ζ_c (solid lines) and the tangent of the half angle of expansion n (dashed lines) are given as functions of ν for Pr = 0.6, 1.0, 1.4, 1.6 (lines 1-4). The boundary of the thermic is determined according to the position of the isocurve $\varphi = 0.1\varphi_{max}$.

For large values of ν, the flow in the thermic is characterized by a small density gradient, and the toroidal vortex is large in size. With a decrease in ν, the vortex becomes more compact, and the intensity of circulatory motion in the medium increases. The surrounding gas captured in the flow deeply penetrates into the "substance" of the thermic, and the lines for equal density are significantly distorted. An increase in Pr leads to an attenuation of the track behind the thermic, and a decrease in ν leads to an amplification of the circulatory motion of the medium.

We will introduce the turbulent analog to the Rayleigh number, which relates the total buoyance reserve of the cloud Π_0, the effective turbulence viscosity coefficients E, and the thermal conductivity E_T: $Ra = \Pi_0/(EE_T) = Pr/\nu^2$. Using self-similarity criteria Ra, the values of ζ_c and n on ν and Pr (Fig. 1) can be reduced to single curves (Fig. 2).

The experimental data in [6] indicate that $\zeta_c \approx 4.35$ for turbulent, axially symmetric thermics, which, according to Fig. 2, corresponds to Ra = 520. The tangent of half the expansion angle of the thermic then has the value $n \approx 0.2$, which is within the range of the experimentally measured quantity.

Hence one can establish a single relation between the effective transfer coefficients $Pr/\nu^2 = 520$ for turbulent thermics, which ensures a correspondence between the calculated and physical flow fields in terms of integral characteristics (the character of the rise of the upper boundary and the expansion angle of the cloud).

The structure of a self-similar turbulent thermic for Pr = 1.6, ν = 0.055 (Ra = 520) is illustrated in Fig. 3, where the solid curves indicate isocurves for φ = const, while the dashed curves are for F = const.

Structures of the thermics calculated for different values of ν and Pr vary for the obtained relation. For obtaining real values of the turbulence coefficient and the Prandtl turbulence number, it is necessary to find additional experimental data on the flow structure.

3. Initial Stage in the Climb of the Thermic. Rendering the functions for calculating the initial stage in the climb of the thermic into dimensionless form, we find that

$$\Omega = \Pi_0^{1/2}R_0^{-2}\widehat{W}(\widehat{\eta}, \widehat{\zeta}, \tau),\ E = \Pi_0^{1/2}\nu,\ \psi = \Pi_0^{1/2}R_0\widehat{F}(\widehat{\eta}, \widehat{\zeta}, \tau),\ \widehat{\eta} = r/R_0,\ \widehat{\zeta} = x/R_0, \quad (3.1)$$
$$\omega = \Pi_0 R_0^{-3}\widehat{\varphi}(\widehat{\eta}, \widehat{\zeta}, \tau),\ \tau = \Pi_0^{1/2}R_0^{-2}t.$$

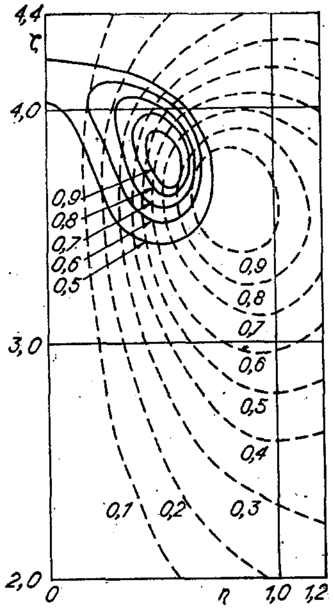

Fig. 3

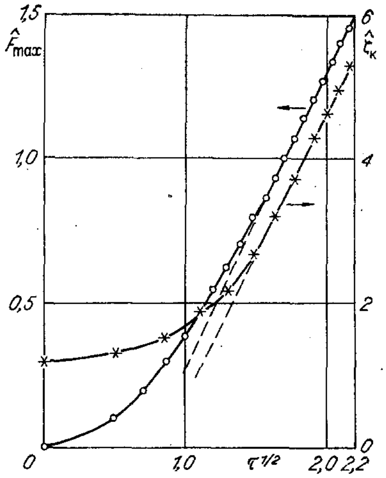

Fig. 4

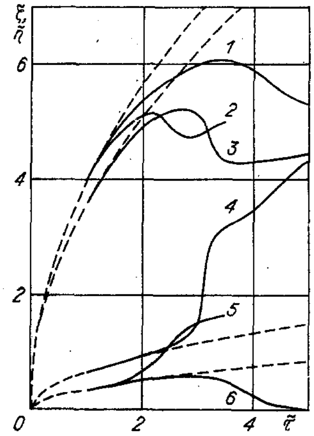

Fig. 5

The dimensionless system of equations obtained after substituting (3.1) into (1.1) for N = 0 is solved in the same manner as in the previous problem for Pr = 1.6, ν = 0.055. The initial conditions are given in the form

$$\tau = 0,\ \hat{W} = \hat{F} = 0,\ \hat{\varphi} = \begin{cases} \hat{\varphi}_0, & \hat{\eta}^2 + (\hat{\zeta} - \hat{\zeta}_0)^2 \leqslant 1, \\ \hat{\varphi}_0 \exp\langle -100\left[\sqrt{(\hat{\zeta} - \hat{\zeta}_0)^2 + \eta^2} - 1\right]^2\rangle, & \hat{\eta}^2 + (\hat{\zeta} - \hat{\zeta}_0)^2 > 1. \end{cases}$$

Here $\hat{\zeta}_0$ is the dimensionless coordinate for the center of the cloud at τ = 0, and the quantity $\hat{\varphi}_0$ is determined from the normalized integral

$$\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty} \hat{\varphi}\hat{\eta} d\hat{\eta} d\hat{\zeta} = 1.$$

Calculations show that, after the volume of buoyant gas is released, a toroidal vortex arises near its lateral boundary whose center is situated at a single height with the center of the stationary cloud. During the evolution of ascending motion, circulation of the gas increases, and the thermic takes on a characteristic mushroom-shaped form. From the time τ = 1.6, the rate of climb for the thermic decreases, and its motion smoothly transfers to the self-similar state. Coordinates for the upper boundary of the thermic $\hat{\zeta}_c$ and the maximum flow function $\hat{F}_{max}$ as functions of the dimensionless time $\tau^{1/2}$ are shown in Fig. 4. Linear dependences correspond to the self-similar state. Using the plot, one can determine the characteristic time of the initial stage in the climb of the buoyant cloud: $0 < t < 3R_0^2\Pi_0^{-1/2}$.

4. Hovering Stage of the Thermic in a Stable Exponential Atmosphere. The case when N = const corresponds to the exponential model of the atmosphere, where a change in the potential temperature Θ over height goes according to $\Theta = \Theta_* \exp(N^2x/g)$ [7]. This model sufficiently describes the real atmosphere, beginning at a height of 1-1.5 km, and is useful for modeling the climb of intense thermics that hover in the tropopause.

We will consider a thermic of thermal nature with an access quantity of heat Q_0 ($\Pi_0 = g\beta Q_0/(2\pi\rho_a c_p)$) relative to the surrounding medium and with a mass M_0 of a passive admixture. Reducing the problem to dimensionless form, we find that

$$\begin{aligned} &\psi = \Pi_0^{3/4} N^{-1/2} \tilde{F}(\tilde{\eta}, \tilde{\zeta}, \tilde{\tau}),\ E = \Pi_0^{1/2}\nu,\ \tilde{\tau} = Nt, \\ &\Omega = N\tilde{W}(\tilde{\eta}, \tilde{\zeta}, \tilde{\tau}),\ \tilde{\eta} = rN^{1/2}\Pi_0^{-1/4},\ \tilde{\zeta} = xN^{1/2}\Pi_0^{-1/4}, \\ &\vartheta = (g\beta)^{-1}\Pi_0^{1/4}N^{3/2}\tilde{\varphi}(\tilde{\eta}, \tilde{\zeta}, \tilde{\tau}), \\ &\varepsilon = M_0\rho_0^{-1}\Pi_0^{-3/4}N^{3/2}\tilde{\varphi}_c(\tilde{\eta}, \tilde{\zeta}, \tilde{\tau}) \end{aligned} \tag{4.1}$$

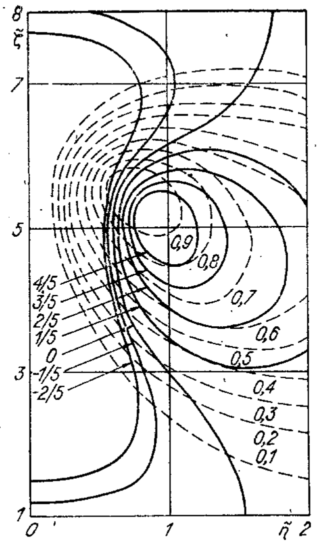

Fig. 6

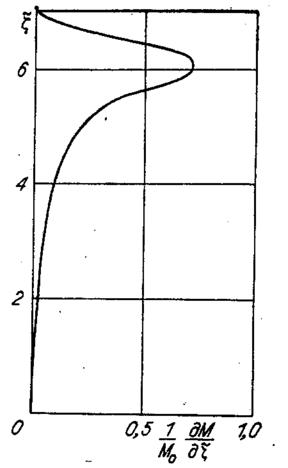

Fig. 7

(ρ_0 is the initial density of the buoyant cloud).

After substituting (4.1) into (1.1), we have

$$\frac{\partial W}{\partial \tau}+\frac{\partial}{\partial \zeta}\frac{W}{\eta}\frac{\partial F}{\partial \eta}-\frac{\partial}{\partial \eta}\frac{W}{\eta}\frac{\partial F}{\partial \zeta}=\nu\left(\frac{\partial^2 W}{\partial \zeta^2}+\frac{\partial^2 W}{\partial \eta^2}+\frac{\partial}{\partial \eta}\frac{W}{\eta}\right)-\frac{\partial \varphi}{\partial \eta}, \tag{4.2}$$

$$\frac{\partial \varphi}{\partial \tau}+\frac{\partial}{\partial \zeta}\frac{\varphi}{\eta}\frac{\partial F}{\partial \eta}-\frac{1}{\eta}\frac{\partial}{\partial \eta}\varphi\frac{\partial F}{\partial \zeta}=\mathrm{Pr}^{-1}\nu\left(\frac{\partial^2 \varphi}{\partial \zeta^2}+\frac{\partial^2 \varphi}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial \varphi}{\partial \eta}\right)-\frac{1}{\eta}\frac{\partial F}{\partial \eta},$$

$$\frac{\partial \varphi_c}{\partial \tau}+\frac{\partial}{\partial \zeta}\frac{\varphi_c}{\eta}\frac{\partial F}{\partial \eta}-\frac{1}{\eta}\frac{\partial}{\partial \eta}\varphi_c\frac{\partial F}{\partial \zeta}=\mathrm{Sc}^{-1}\nu\left(\frac{\partial^2 \varphi_c}{\partial \zeta^2}+\frac{\partial^2 \varphi_c}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial \varphi_c}{\partial \eta}\right),$$

$$W=\frac{1}{\eta^2}\frac{\partial F}{\partial \eta}-\frac{1}{\eta}\frac{\partial^2 F}{\partial \eta^2}-\frac{1}{\eta}\frac{\partial^2 F}{\partial \zeta^2},\quad \int_{-\infty}^{\infty}\int_{0}^{\infty}\varphi_c \eta\, d\eta\, d\zeta=1,$$

$$W=F=\frac{\partial \varphi}{\partial \eta}=\frac{\partial \varphi_c}{\partial \eta}=0,\ \eta=0,\ W=F=\varphi=\varphi_c\to 0,\ \eta^2+\zeta^2\to\infty$$

(the tilde is omitted in this system of equations).

For calculating the dynamics of a hovering thermic, we will use the following self-similar solution as initial conditions

$$\begin{gathered}\tilde{W}(\tilde{\eta},\tilde{\zeta},\tilde{\tau}_0)=\tilde{\tau}_0^{-1}W(\eta,\zeta),\quad \tilde{\eta}=\tilde{\tau}_0^{1/2}\eta,\quad \tilde{\zeta}=\tilde{\tau}_0^{1/2}\zeta,\\ \tilde{F}(\tilde{\eta},\tilde{\zeta},\tilde{\tau}_0)=\tilde{\tau}_0^{1/2}F(\eta,\zeta),\quad \eta=r\Pi_0^{-1/4}t^{-1/2},\quad \zeta=x\Pi_0^{-1/4}t^{-1/2},\\ \tilde{\varphi}(\tilde{\eta},\tilde{\zeta},\tilde{\tau}_0)=\tilde{\varphi}_c(\tilde{\eta},\tilde{\zeta},\tilde{\tau}_0)=\tilde{\tau}_0^{-3/2}\varphi(\eta,\zeta).\end{gathered} \tag{4.3}$$

Since the solution must not be a function of $\tilde{\tau}_0$, it is determined through inspection from the condition that the flow remains in the self-similar state for some time after $\tilde{\tau}_0$.

The dynamics of a hovering thermic for Pr = Sc = 1.6, ν = 0.055, $\tilde{\tau}_0$ = 1 are indicated in Fig. 5 by the dimensionless vertical and radial coordinates of the characteristic points of the cloud [curves 1 and 6 – the maxima of the admixture concentration ($\tilde{\zeta}_C$, $\tilde{\eta}_C$); curves 2 and 5 – the excess temperature ($\tilde{\zeta}_T$, $\tilde{\eta}_T$); and curves 3 and 4 – the flow function ($\tilde{\zeta}_F$, $\tilde{\eta}_F$)] as functions of the dimensionless time $\tilde{\tau}$, where the dashed lines correspond to the self-similar characteristics. It is evident that the thermic "descends" from the self-similar state after some time ($\tilde{\tau} > \tilde{\tau}_0$). The rate of climb decreases while the radius increases. When $\tilde{\tau} \approx 2.6$ and $\tilde{\tau} \approx 3.4$, the vortex center and the point of maximum admixture concentration are at their greatest heights, respectively:

$$x_F \approx 5{,}2\Pi_0^{1/4}N^{-1/2},\quad x_c \approx 6{,}1\Pi_0^{1/4}N^{-1/2}.$$

After passing the level of thermal equilibrium, the cloud begins to oscillate. When the excess thermal energy of the thermic becomes small, the climb of the gas particles in the

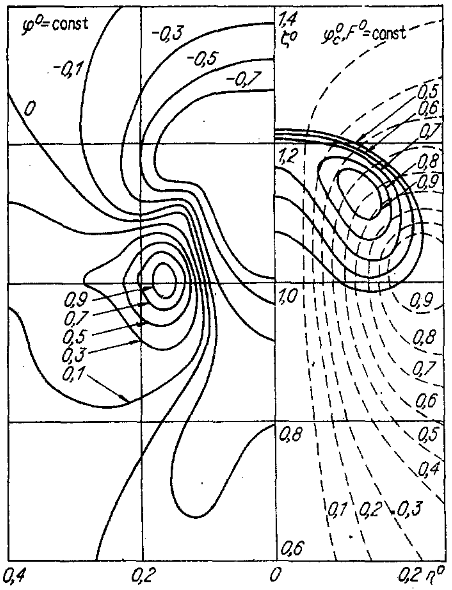

Fig. 8

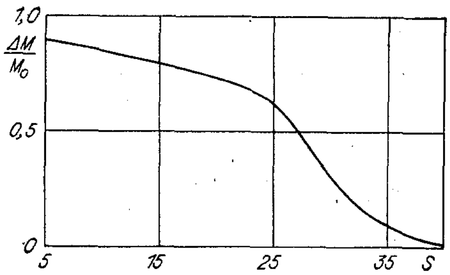

Fig. 9

TABLE 1

s	ζ_F^0	ζ_c^0
56,00	0,73	0,81
43,38	0,79	0,93
32,33	0,88	1,02
25,05	0,96	1,12
20,00	1,05	1,20
13,72	1,14	1,32
6,13	1,60	1,83

toroidal vortex results in their overcooling, while descent entails overheating. Then, a region of overcooling relative to the surrounding medium is formed near the axis of symmetry for convective formation. A region of insignificant overheating is located at the periphery. The structure of the calculated flow at time $\tilde{\tau} = 2.6$ is given in Fig. 6, where the solid lines represent isotherms and the dashed lines indicate lines for equivalent flow functions. The numbers near the curves are values of the functions on isocurves in relation to their maximum values. The toroidal vortex generated by the climbing buoyant cloud rapidly increases in transverse dimension at the hovering stage, but the velocity of the gas in it and the convective transfer of the admixture decrease. The region of maximum admixture concentration approximates the axis of symmetry over time, and the isocurves for the concentration become almost spherical in form.

The distribution of the admixture introduced by the cloud over the height of the atmosphere at the time $\tilde{\tau} = 3.4$ is shown in Fig. 7 and was obtained by integrating the concentration field over the radius.

5. Transfer of the Thermic into the Lower Layers of the Stratosphere. A sufficiently intense thermic (clouds from nuclear explosions or volcanic eruptions) can climb to heights greater than the tropopause and enter the lower layers of the stratosphere. For modeling this case, one must account for the different states of the atmosphere up to and above the tropopause. According to the international standard model of the atmosphere, the Väisälä–Brunt coefficient in the troposphere has the value $N_1 = 0.011$ sec^{-1}. If $x = x_T$, then N changes abruptly up to $N_2 = 0.021$ sec^{-1}, which is the case in the lower layers of the stratosphere. The position of the level of the tropopause x_T varies with latitude and season. For the northern hemisphere, $x_T = 10$ km, on the average, while it is 16 km in the southern hemisphere.

The problem of a climbing thermic in a two-layer atmosphere can be put into dimensionless form as follows:

$$\Omega = \Pi_0^{1/2} x_T^{-2} W^0(\eta^0, \zeta^0, \tau^0), \quad E = \Pi_0^{1/2}\nu,$$
$$\psi = \Pi_0^{1/2} x_T F^0(\eta^0, \zeta^0, \tau^0), \quad \tau^0 = \Pi_0^{1/2} x_T^{-2} t,$$
$$\vartheta = (g\beta)^{-1}\Pi_0 x_T^{-3}\varphi^0(\eta^0, \zeta^0, \tau^0), \quad \eta^0 = r/x_T,$$
$$\varepsilon = M_0\rho_0^{-1} x_T^{-3}\varphi_c^0(\eta^0, \zeta^0, \tau^0), \quad \zeta^0 = x/x_T,$$

The dimensionless system of equations is similar to (4.2) with the exclusion of the last term in the equation for $\tilde{\varphi}$, which is written in the form $-1/\eta^0 \partial F^0/\partial\eta^0 S^2 f(\zeta^0)$. Here, $S = N_1 x_T \Pi_0^{-1/2}$ is a dimensionless parameter that is the ratio of the squares of the height of the tropopause to the characteristic height for hovering of the cloud in the troposphere. Using the indicated model of the atmosphere, $f(\zeta^0) = 1$ when $\zeta^0 \leq 1$ and $f(\zeta^0) = N_2/N_1 = 1.91$ when $\zeta^0 > 1$.

As an initial condition for $\tau^0 = \tau_0{}^0$, we will use the self-similar solution for $Pr = Sc = 1.6$ and $\nu = 0.055$. Recalculation of the initial distributions for the dimensionless density functions is similar to that for (4.3). The calculations were done for different values of the parameter S. The obtained maximum heights for the climb of characteristic points in the cloud — the vortex center $\zeta_F{}^0$ and the maximum admixture concentration $\zeta_c{}^0$ — are given in Table 1.

When $S > 56$, the cloud hovers in the troposphere, while for $S < 10$, the climb deviates from the self-similar state in the stratosphere. The solutions for $S > 56$ and <10 coincide with those for a single-layer atmospheric model if one assumed that $N = N_1$ for the first case and $N = N_2$ for the second case. Hence, for the indicate range of values for S, the thermic does not "feel" the abrupt transition to the stratosphere at the hovering stage and can be described using the single-layer atmospheric model.

The flow structure for an intermediate value of $S = 20$ at time $\tau^0 = 0.084$ is illustrated in Fig. 8. The lower layers of the stratosphere suppress the climbing convective flow of the gas more intensely than the troposphere ($N_2 > N_1$). This flattens the isotherms of the cloud upon transition through the tropopause. If $\tau^0 < 0.06$, the climb of the cloud for a given value of S occurs in the self-similar state, and the flow structure in it is similar to that in Fig. 3. If $\tau^0 > 0.3$, the cloud becomes spheroidal.

The above results for the hovering stage of the thermic in a two-layer atmosphere allow one to determine the fraction of the admixture introduced into the stratosphere by the cloud as a function of the parameter S (Fig. 9). This dependence can be used to estimate the quantity of admixture ejected beyond the tropopause for various positions of the level x_T and to determine the thermal energy of the thermic. Hence, for $x_T = 10$ km, the thermal energy of the cloud Q (J) corresponds to the quantity [2]

$$S = \frac{x_T^2 N_1}{[g\beta Q/(2\pi\rho_a c_p)]^{1/2}} \approx 5.3\cdot 10^8 Q^{-1/2}.$$

Then, according to Fig. 9, a thermic with an energy less than $1.5\cdot 10^{14}$ J hovers below the tropopause and does not eject the admixture into the stratosphere. In the southern hemisphere, contamination of the stratosphere by an admixture contained in a buoyant cloud will not occur if the thermal energy of the cloud does not exceed $1.04\cdot 10^{15}$ J.

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THERMAL GRAVITATIONAL CONVECTION IN A VARIABLE VECTOR FIELD OF SMALL ACCELERATIONS

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The use of nearly weightless states in the manufacture of materials can allow for improvement of structure and for uniformity of mixture distribution in samples [1-3]. In the absence of gravity, small accelerations due to various perturbations play a primary role in the evolution of gravitational convection. Small accelerations are related to the rigidity characteristics of structures and are periodic in nature, where the vector for small accelerations **g** continuously changes in quantity and direction over time. In many cases, this change can to some degree of accuracy be considered as the rotation of a vector with a constant modulus and angular velocity in some fixed plane

$$|\mathbf{g}| = \mathrm{const},\ \theta_g = \tilde{\omega}\tilde{t}, \tag{1}$$

where $\tilde{\omega}$ is the angular velocity of rotation; θ_g is the angle between the current and initial directions of the vector **g**; and the symbol ~ will be used to denote dimensional quantities.

In order to see what effect (and if there is an effect, in what manner) a change in the vector of a small local acceleration has on the evolution of convective transfer processes, we studied the model problem of thermal gravitational convection in a cylindrical volume with rotation **g** in a plane perpendicular to the axis of the cylinder.

The mathematical model for the calculation scheme is given in Fig. 1, where one must consider the transfer equations for momentum and energy in the variables T, ψ, w (the temperature, the flow function, and the vortex intensity function) and the equation for the relation between ψ and w. Using the polar coordinate system in dimensionless form with the Boussinesq approximation, these equations have the form: form the momentum transfer equation,

$$\frac{\partial w}{\partial \mathrm{Fo}} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} = \mathrm{Pr}^2\mathrm{Gr}\left\{\frac{\partial T}{\partial r}\sin(\theta - \theta_g) + \frac{\partial T}{\partial \theta}\frac{\cos(\theta - \theta_g)}{r}\right\} + \frac{\mathrm{Pr}}{r^2}\left\{r\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial \theta^2}\right\};$$

for the energy transfer equation,

$$\frac{\partial T}{\partial \mathrm{Fo}} + u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2},$$

$$u = \frac{1}{r}\frac{\partial \psi}{\partial \theta},\quad v = -\frac{\partial \psi}{\partial r},$$

and for the equation for the relation between ψ and w,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = w,$$

while the change in **g** over time is given by relation (1). Conventional definitions are used here, and the transformation to dimensionless quantities is done with the relations

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